Tutorial Sheet No.1

Topics Analytic functions **Course: complex variables** **Semester & Branch: 5th ECE**

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Q.1 Show that the function $v = k\left(r - \frac{a^2}{r}\right) Cos\theta$ is harmonic. Determine its harmonic conjugate and the corresponding analytic function $f(z) = u + iv$.

Q.2 Determine whether each of the following statements is true or false. Justify your answer with a proof or a counter example.

(i) An entire function f such that $f(x+iy) = u(x) + iv(y)$ must be of the form $f(z) = \alpha z + \beta$ with $\alpha \in R$ and $\beta \in C$.

(ii) There exists no analytic function f such that $Ref(z) = y^2 - 2x$.

(iii) An analytic function $f = u + iv$ in a domain D such that $v = u^2$ is necessarily a constant.

(iv) The law of trichotomy does not hold good in the set of complex numbers.

Q.3 For the function defined by $f(x) = \sqrt{|xy|}$, Examine whether that the Cauchy Riemann equations are satisfied at origin. What can you say about the analyticity of the function at origin?

Q.4 Prove that $\left|\frac{z-1}{z+1}\right| = const.$ and $amp\left(\frac{z-1}{z+1}\right) = const.$ are orthogonal circles.

Q.5 Let $f(z) = u + iv$ and $g(z) = v + iu$ be analytic functions for all z. Let $f(0) = 1$ and $g(0) = i$. Obtain the values of $h(z)$ at $z = 1 + i$, where $h(z) = f'(z) + g'(z) + 2\, f(z)g(z)$.

Q.6 Determine whether each of the following statements is true or false. Justify your answer with a proof or a counter example.

(i) If the function $w = f(z) = u(r,\theta) + i\, v(r,\theta)$ is analytic then $\frac{dw}{dz} = e^{-i\theta}\frac{\partial w}{\partial r}$.

(ii) There exists no analytic function f such that $Re[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$.

(iii) Ordering property holds good for complex numbers.

(iv) If the function $f(z)$ and $\overline{f(z)}$ are both analytic, then $f(z)$ is a constant.

Q.7. For what values of $'a'$ and $'b'$ the function $f(z) = (1+ib)x^2 + (a-i)y^2 - 2(1-i)xy$ is analytic. Also find $f'(z)$.

Q.8 Give an example to show that (i) Arg $(z_1 z_2) \neq Arg(z_1) + Arg(z_2)$

(ii) arg $(z_1 z_2) = arg(z_1) + arg(z_2)$