Course on Relativity: Dr P A Ganai

Ist Semester Btech Batch 2016: Unit 3rd General Physics Phy101

****

Reference Books and Notes:

1. Modern Physics by P A Tipler 5th edition W. H. Freeman and Company New York
2. Relativity: The Special and General Theory by Albert Einstein (available free on-line)
3. Special Relativity by David W. Hogg(Institute for Advanced Study Olden Lane Princeton)
4.ON THE ELECTRODYNAMICS OF MOVING BODIES By A. EINSTEIN
This is the original research paper in which the idea of relativity was proposed. The paper is available for free download.
5.Lecture Notes on Special Relativity by J D Cresser
Department of Physics Macquarie University

Video lectures and web resources

1.https://ocw.mit.edu/courses/physics/8-033-relativity-fall-2006/download-course-materials/
2.https://www.youtube.com/watchv=0nHovWsWZTw&list=PLkmCRrIU3CbsiGwBFaVTUAUWesfEg9sku
3.https://www.youtube.com/watch?v=toGH5BdgRZ4
4 https://www.youtube.com/watch?v=ng6ANMGNlpg&list=PL9jo2wQj1WCPvSAfQkKODyJo47d7tyXMr

Introduction: Special theory of Relativity?

Until the end of 19th century Newtonian mechanics was supposed to be basic frame work behind every natural phenomena. To describe any event , three space coordinates are assigned to lactate position of the particle and time is believed to be absolute (Running in one direction at constant rate).

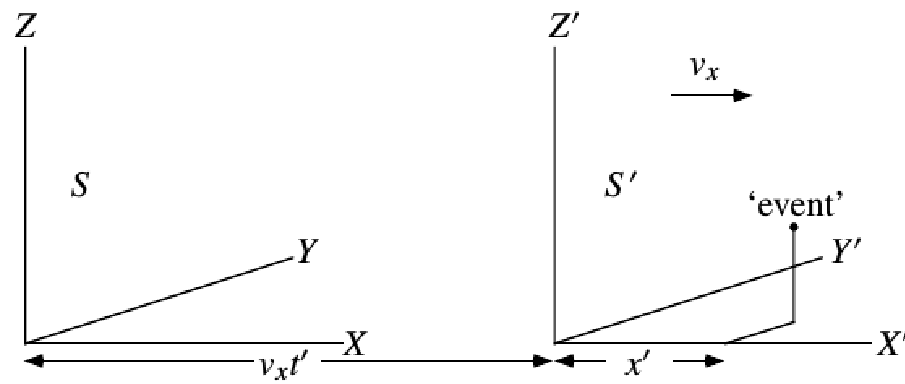
However, with the formulation of unified theory of electromagnetism disrupted this comfortable state of affairs – the theory was extraordinarily successful, yet at a fundamental level it seemed to be inconsistent with certain aspects of the Newtonian ideas of space and time. Ultimately, a radical modification of these latter concepts, and consequently of Newton's equations themselves, was found to be necessary. It was Albert Einstein who, by combining the experimental results and physical arguments of others with his own unique insights, first formulated the new principles in terms of which space, time, matter and energy. These principles, and their consequences constitute the Special Theory of Relativity.

Newtonian Relativity:

The simple question that one can ask is that are laws of motion dependent on who is making measurements. In the frame work of Newtonian mechanics , it turns out that the form of laws is same for observers who are either at rest or are moving with constant velocity. Such observes are called inertial observes or inertial frames. It was Galileo who first introduced the concept of inertia and the same concept was used by Newtonian in his theory of Mechanics. The measurements of one fame can be related with other frame though transformation equations are Galilean Transformations. Lets try to understand mathematical frame work of Galilean transformations

Galilean Transformations

To derive these transformation equations, consider an inertial frame of reference S and a second reference frame S' moving with a velocity ' v' relative to S .



Let us suppose that the clocks in S and S ' are set such that when the origins of the two reference frames O and O' coincide, all the clocks in both frames of reference read zero i.e. t = t ' = 0. According to 'common sense', if the clocks in S and S ' are synchronized at t = t' = 0, then they will always read the same, i.e. t = t' always. Suppose now that an event of some kind, e.g. an explosion, occurs at a point (x' , y' , z', t') according to S' . Then, by examining above figure, according to S , it occurs at the point

$$x = x' + v_x t', \quad y = y', \quad z = z'$$
$$t = t'$$

These equations together are known as the Galilean Transformation, and they tell us how the coordinates of an event in one inertial frame S are related to the coordinates of the same event as measured in another frame S ' moving with a constant velocity relative to S . Now suppose that in inertial frame S , a particle is acted on by no forces and hence is moving along the straight line path given by:

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t$$

where u is the velocity of the particle as measured in S . Then in S' , a frame of reference moving with a velocity **v** relative to S , the particle will be following a path

$$\mathbf{r}' = \mathbf{r}_0 + (\mathbf{u} - \mathbf{v})t'$$

where we have simply substituted for the components of **r .** This last result also obviously represents the particle moving in a straight line path at constant speed. And since the particle is being acted on by no forces, S ' is also an inertial frame, and since v is arbitrary, there is in general an infinite number of such frames.

Incidentally, if we take the derivative of above equation with respect to t, and use the fact that t = t ', we obtain

$$\mathbf{u}' = \mathbf{u} - \mathbf{v}$$

which is the familiar addition law for relative velocities.
It is a good exercise to see how the inverse transformation can be obtained from the above equations.

$$\left.\begin{aligned} x' &= x - v_x t \\ y' &= y \\ z' &= z \\ t' &= t. \end{aligned}\right\}$$

Newtonian Relativity

By means of the Galilean Transformation, we can obtain an important result of Newtonian mechanics which carries over in a much more general form to special relativity. We shall illustrate the idea by means of an example involving two particles connected by a spring. If the X coordinates of the two particles are x1 and x2 relative to some reference frame S then from Newton's Second Law the equation of motion of the particle at x1 is

$$m_1 \frac{d^2 x_1}{dt^2} = -k(x_1 - x_2 - l)$$

where ‘k’ is the spring constant, ‘l’ the natural length of the spring, and m1 the mass of the particle. If we now consider the same pair of masses from the point of view of another frame of reference S’ moving with a velocity v x relative to S , then

$$x_1 = x'_1 + v_x t' \quad \text{and} \quad x_2 = x'_2 + v_x t'$$

$$\frac{d^2 x_1}{dt^2} = \frac{d^2 x'_1}{dt'^2}$$

and

$$x_2 - x_1 = x'_2 - x'_1.$$

Thus, substituting the last two results

$$m_1 \frac{d^2 x'_1}{dt'^2} = -k(x'_1 - x'_2 - l)$$

Now according to Newtonian mechanics, the mass of the particle is the same in both frames i.e.

$$m_1 = m'_1$$

where m’ is the mass of the particle as measured in S ‘ . Hence

$$m'_1 \frac{d^2 x'_1}{dt'^2} = -k(x'_1 - x'_2 - l)$$

which is exactly the same equation as obtained in S , except that the variables x1 and x2 are replaced by x1’ and x2’ . In other words, the form of the equation of motion derived from Newton’s Second Law is the same in both frames of reference. This result can be proved in a more general way than for than just masses on springs, and we are lead to conclude that the mathematical form of the equations of motion obtained from Newton’s Second Law are the same in all inertial frames of reference.

Continuing with this example, we can also show that momentum is conserved in all inertial reference frames. Thus, in reference frame S , the total momentum is

$$m_1\dot{x}_1 + m_2\dot{x}_2 = P = \text{ constant.}$$

Then see that in S the total momentum is

$$P' = m_1'\dot{x}_1' + m_2'\dot{x}_2' = m_1\dot{x}_1 + m_2\dot{x}_2 - (m_1 + m_2)v_x = P - (m_1 + m_2)v_x$$

which is also a constant (but not the same constant as in S it is not required to be the same constant!!). The analogous result to this in special relativity plays a very central role in setting up the description of the dynamics of a system. The general conclusion we can draw from all this is that:

Newton’s Laws of motion are identical in all inertial frames of reference.

This is the Newtonian (or Galilean) principle of relativity, and was essentially accepted by all physicists, at least until the time when Maxwell put together his famous set of equations. One consequence of this conclusion is that it is not possible to determine whether or not a frame of reference is in a state of motion by any experiment involving Newton’s Laws. At no stage do the Laws depend on the velocity of a frame of reference relative to anything else, even though Newton had postulated the existence of some kind of ”absolute space” i.e. a frame of reference which defined the state of absolute rest, and with respect to which the motion of anything could be measured. The existence of such a reference frame was taken for granted by most physicists, and for a while it was thought to be have been uncovered following on from the appearance on the scene of Maxwell’s theory of electromagnetism.

Maxwell's Equations and the Ether:

The Newtonian principle of relativity had a successful career till the advent of Maxwell's work in which he formulated a mathematical theory of electromagnetism which, amongst other things, provided a successful physical theory of light. Not unexpectedly, it was anticipated that the equations Maxwell derived should also obey the above Newtonian principle of relativity in the sense that Maxwell's equations should also be the same in all inertial frames of reference. Unfortunately, it was found that this was not the case. Maxwell's equations were found to assume completely different forms in different inertial frames of reference. It was as if F = ma worked in one frame of reference, but in another, the law had to be replaced by some bizarre equation like $F' = m(a')^2 a'$! In other words it appeared as if Maxwell's equations took a particularly simple form in one special frame of reference, but a quite complicated form in another moving relative to this special reference frame. For instance, the wave equation for light assumed the simple form

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

in this 'special frame' S , which is the equation for waves moving at the speed c. Under the Galilean transformation, this equation becomes

$$\frac{\partial^2 E'}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E'}{\partial t'^2} - \frac{2v_x}{c^2}\frac{\partial^2 E'}{\partial x' \partial t'} - \frac{v_x}{c^2}\frac{\partial}{\partial x'}\left[v_x \frac{\partial E'}{\partial x'}\right] = 0$$

for a frame S' moving with velocity v_x relative to S . This 'special frame' S was assumed to be the one that defined the state of absolute rest as postulated by Newton, and that stationary relative to it was a most unusual entity, the ether. The ether was a substance that was supposedly the medium in which light waves were transmitted in a way something like the way in which air carries sound waves. Consequently it was believed that the behavior of light, in particular its velocity, as measured from a frame of reference moving relative to the ether would be different from its behavior as measured from a frame of reference stationary with respect to the ether.

Since the earth is following a roughly circular orbit around the sun, then it follows that a frame of reference attached to the earth must at some stage in its orbit be moving relative to the ether, and hence a change in the velocity of light should be observable at some time during the year. From this, it should be possible to determine the velocity of the earth relative to the ether. An attempt was made to measure this velocity. This was the famous experiment of Michelson and Morley. Simply stated, they argued that if light is moving with a velocity c through the ether, and the Earth was at some stage in its orbit moving with a velocity v relative to the ether, then light should be observed to be traveling with a velocity c' = c – v relative to the Earth. We can see this by simply solving the wave equation in S :

$$E(x,t) = E(x - ct)$$

where we are supposing that the wave is traveling in the positive X direction. If we suppose the Earth is also traveling in this direction with a speed v_x relative to the ether, and we now apply the Galilean Transformation to this expression, we get, for the field E' (x' , t ') as measured in S ',, the result

$$E'(x', t') = E(x, t) = E(x' + v_x t' - ct') = E(x' - (c - v_x)t')$$

i.e. the wave is moving with a speed $c - v_x$ which is just the Galilean Law for the addition of velocities .

Needless to say, on performing their experiment – which was extremely accurate – they found that the speed of light was always the same. Obviously something was seriously wrong. Their experiments seemed to say that the earth was not moving relative to the ether, which was manifestly wrong since the earth was moving in a circular path around the sun, so at some stage it had to be moving relative to the ether. Many attempts were made to patch things up while still retaining the same Newtonian ideas of space and time. Amongst other things, it was suggested that the earth dragged the ether in its immediate vicinity along with it. It was also proposed that objects contracted in length along the direction parallel to the direction of motion of the object relative to the ether. This suggestion, due to Fitzgerald and elaborated on by Lorentz and hence known as the Lorentz-Fitzgerald contraction, 'explained' the negative results of the Michelson-Morley experiment, but faltered in part because no physical mechanism could be discerned that would be responsible for the contraction. The Lorentz-Fitzgerald contraction was to resurface with a new interpretation following from the work of Einstein. Thus some momentary successes were achieved, but eventually all these attempts were found to be unsatisfactory in various ways. It was Einstein who pointed the way out of the impasse, a way out that required a massive revision of our concepts of space, and more particularly, of time.

****