Dynamics of STructures (CSE-102)

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COURSE CONTENTS

S. No.	Contents	Contact Hours
1	Nature of dynamic loading: Harmonic, earthquake and blast loading, Single degree of freedom systems, free vibrations and forced vibrations: Harmonic force, Periodic force, Impulse, and General type of loading.	6

REFERENCES

S. No.	Name of Books/Authors/Publishers
1	Dynamics of Structures By Anil K. Chopra
2	Dynamics of Structures By Clough and Penzien
3	Structural Dynamics By Mario Paz

Lecture 1st

1.1 Introduction

Structural dynamics is a type of structural analysis which covers the behavior of structures subjected to dynamic loadings. Dynamic loadings are actions having high acceleration. Dynamic loads include earthquake and blasts.

Dynamic analysis can be used to find dynamic displacements, time history and model analysis.

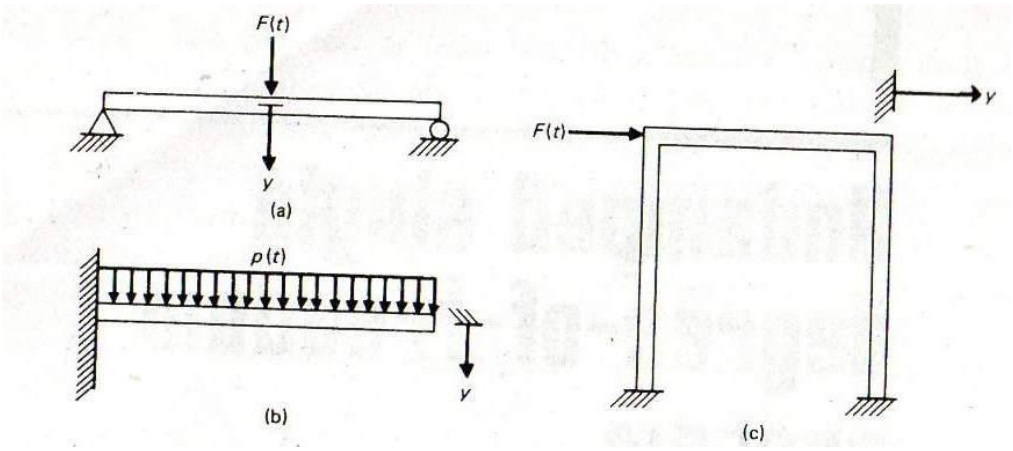
A dynamic analysis is also related to the initial forces developed by the structure when its excited by means of dynamics loads applied suddenly e.g. Wind blasts, explosions and earthquake.

1.1.1 Degree of Freedom:-when a structure is loaded, specified points on it will undergo unknown displacement. These displacements are referred to a degree of freedom for structure.

In 3D each node on a frame or beam can have at most 3 linear and 3 rotational displacements.

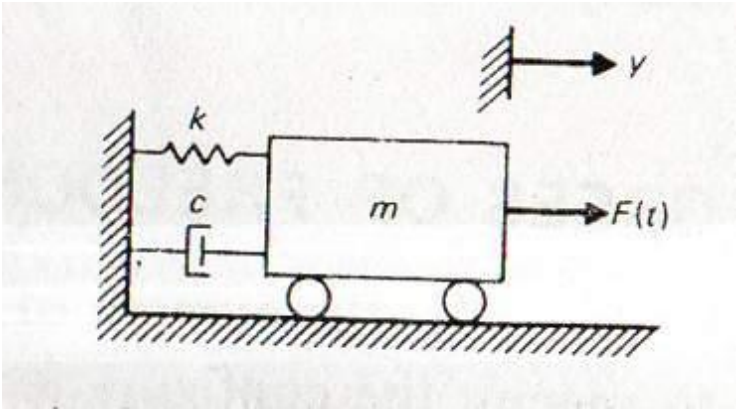
In 2D each node can have 2 linear displacements and one rotational displacement.

The process of idealization or selection of appropriate mathematical to model permits the reduction in the No. of degree of freedom to discrete No. and in some cases to just single degree of freedom.



The structures shown above may be represented for dynamic analysis as one degree of freedom system. ie " Structures Modeled as system with a single displacement coordinate".

Mathematical Model for one degree of freedom system.



m- Mass element 'm' representing Mass and inertial characteristics of structure.

k- Spring element 'k' representing the elastic restoring force and potential energy capacity of structure.

C:- Damping 'C' representing the frictional characteristics and energy loses of structure.

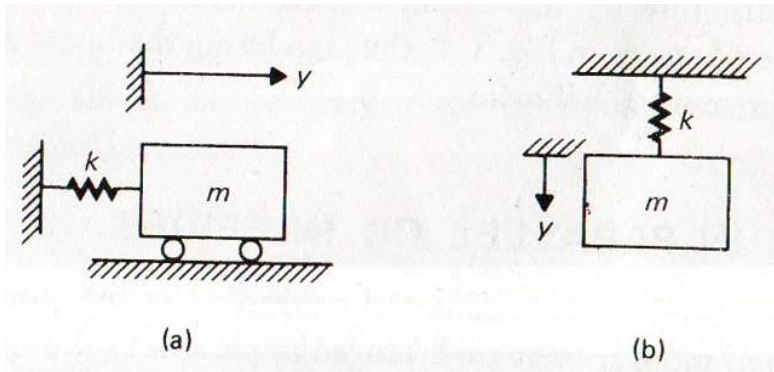
F(t) – Excitation force representing external force acting on structure F (t) is written in this way to represent it as function of time.

Every element in Mathematical Model represents single property.

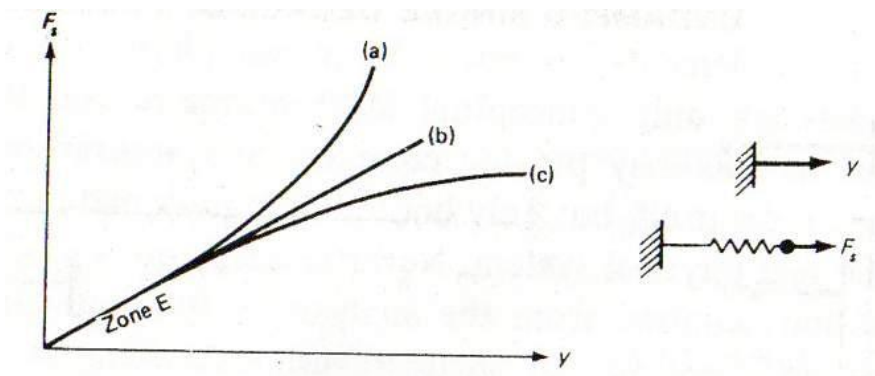
1.2 Undamped system:

Let's start our study of structural dynamics with the analysis of fundamental and simple system, ie one degree of freedom system in which we neglect frictional force and vibration force from external forces under these conditions, the system is in motion governed only by the influence of the so called initial conditions ie given displacement and velocity at time t=0. This undamped one degree of freedom system is often referred to as simple undamped oscillator.

Representation



In the Models shown above the Mass m is restrained by spring k and limited to rectilinear Motion along one coordinate axis. The mechanical characteristics of a spring is described by a relation between the magnitude of force F (s) applied to its free end the resulting end displacement y as shown graphically in Fig. below for three different springs.



a. (Hard Spring)
b. (Linear Spring)
c. (soft Spring)

Hard Spring:- Force required to produce a given displacement becomes increasing greater as spring deformed. Soft spring is vice versa from linear spring.

From linear spring

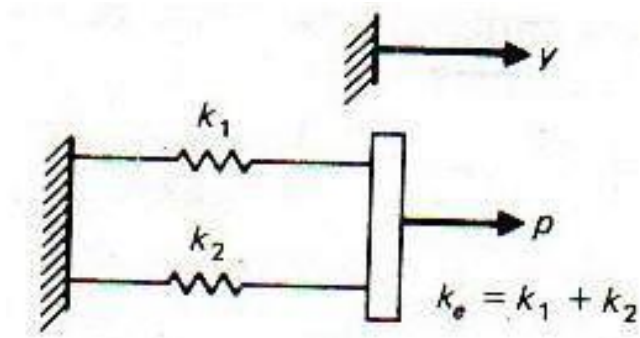
Base for our dynamics works/analysis.

$F_s \alpha y$

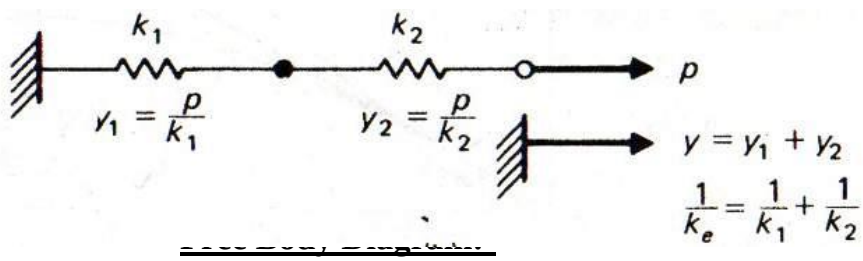
$F_s = k\,y$ where k = Spring constant

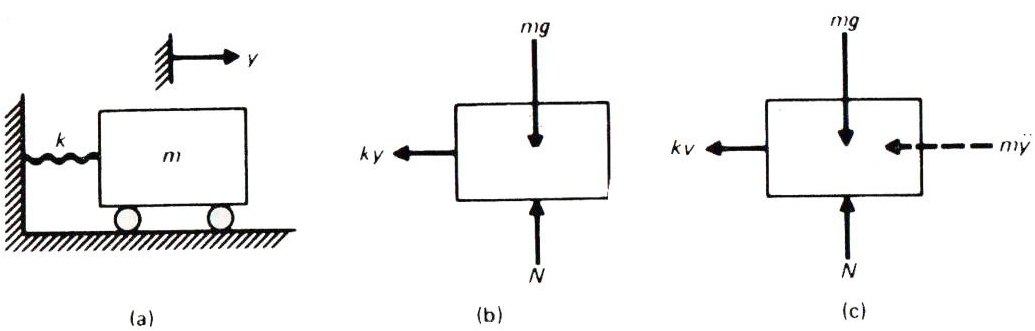
Zone E represents displacements produced in structure by the action of external force or disturbances of small Magnitude, thus rendering linear approximately close to structural behavior.

1.3.1 Spring in series or parallel:-



1.3.2 Springs in series:-





Single degree of freedom system (a)

External forces only (b)

External and Internal forces (c)

The F BD is a sketch of the body isolated from all other bodies, in which all the forces external to the body are shown. As shown in Fig. (b) F BD of Mass m of the oscillator displaced in positive direction wrt to coordinate y and acted upon the spring force F_s=ky

(assuming linear). The weight of body mg and normal reaction N of the supporting surface are also shown for completeness. Though these forces act in virtual direction do not enter into the equation of Motion written for y direction. The application of Newton's Law of Motion gives.

$F = Ma$ $\qquad$ y = displacement

$-ky = m\ddot{y}$ $\qquad$ $\acute{y}$ = velocity

$\ddot{y}$ = acceleration

Where spring force acts in negative direction/opposite so it has negative sign.

1.4 D' Alembert's Principle:-

An alternative approach to obtain eq $-k\ y = m\ \ddot{y}$ is the make use of D Alembert's Principle, which states that a system may be set in a state of dynamic equilibrium by adding to the external forces a fictitious force which is commonly known as inertial force.

The application of D Alembert's Principle allows us to use eqn of eqmb in obtaining eq of Motion.

$\Sigma F_y = m\ \ddot{y} + k\ y = 0$

Or $-ky = m\ \ddot{y}$.

<u>Solution of the differential Eq of Motion</u>

$(m\ \ddot{y} + k\ y = 0)$ ----- (1)

Let's adopt a systematic approach and proceed to first classify the differential Eq. Since dependent variable $\acute{y}$ and $\ddot{y}$ appear in the first degree so this eq is classified as linear and of second order.

The fact that the coefficients of $\acute{y}$ and $\ddot{y}$ are k and m respectively are constants and r h s of eq is zero.

Further classify the eq as homogeneous with constant coefficients.

For this simple second order differential eq we may proceed directly by assuming a trial solution given by

$y = A \cos \omega t$ ------ (2)

$y = B \sin \omega t$ ------- (3)

Where A and B are constants depending on imitation of Motion.

ω is quantity denoting physical characteristics of system

Subt eq 2 in eq 1

$(m \omega^2 + k) A \cos \omega t = 0$ ------- (4)

This eqn is to be satisfied at any time the factor in parentheses must be equal to zero or

$\omega^2 = k/m$ ------- (5)

eq 3 is also solution of differential eq 1 with ω also satisfying (5)

$\omega = \sqrt{k/m}$ (6)

→ is known as natural frequency of system

Eqn 2 and 3 are solution of 1 since it's a differential eq and linear supper position of these two solutions is also a solution.

$y = A \cos \omega t + B \sin \omega t$ ------- (7)

$\acute{y} = -A \omega \sin \omega t + B \omega \cos \omega t$ ------ (8)

Now we will find values of integration constants A and B using initial conditions.

When $t = 0$, $y = y_0$ and $y = v_0$

Subt in eq 7 and 8

$y_0 = A$ ------ (9)

$v_0 = B\omega$ ------ (10)

Finally $y = y_0 \cos \omega t + v_0/\omega \sin \omega t$ ------ (11)

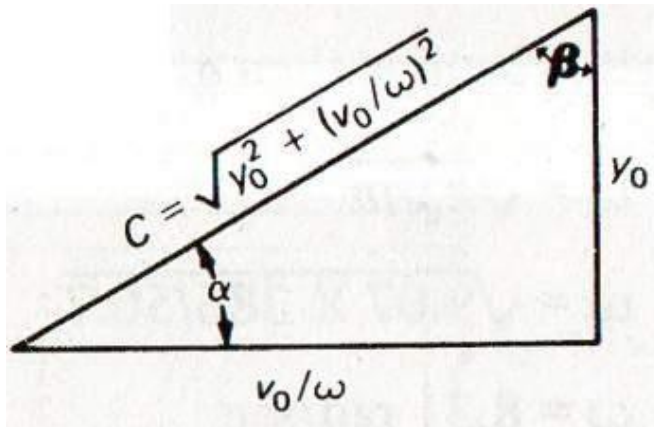
Thus the Motion of simple undamped oscillator Modeling structures with a single degree of freedom is given by y or eq 11

1.5 Amplitude of Motion:

Let's now examine in more detail eq 11. The simple trigonometric transformation may show us that we rewrite eq in equalent forms namely:

$y = (\sin(\omega t + \alpha)$ ------ (12)

$y = (\cos(\omega t - \beta)$ ------ (13)



$C = \sqrt{y_0^2 + (v_0/\omega)^2}$ ----- (14)

$\tan\alpha = y_0 / v_0/\omega$ ------- (15)

and $\tan\beta = v_0/\omega/y$ ------ (16)

or

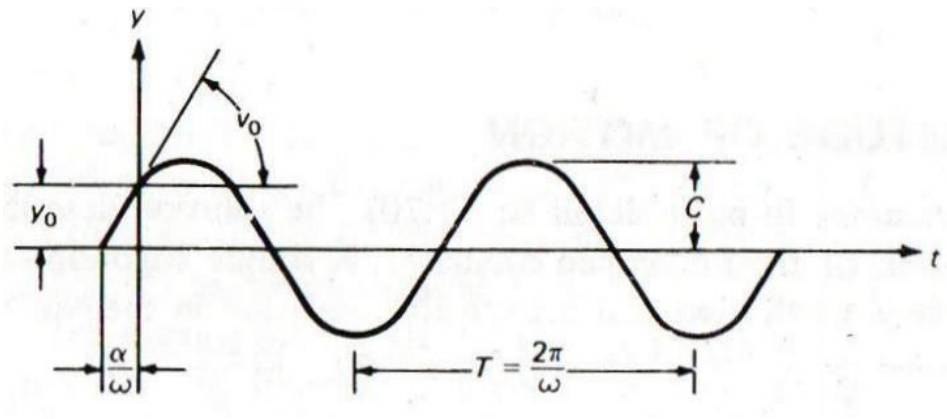
Multiple and divide eq 11 by factor C ie eq 14 to define α or β by eq 15 and 16

$y = C\,[y_0/C \cos\omega t + v_0/\omega/C \sin\omega t]$

$\sin\alpha = y_0/C$ and $\cos\alpha = v_0/\omega$

$y = C\,[\sin\alpha \cos\omega t + \cos\alpha \sin\omega t]$ ------ (17)

The value of C in eq $C = \sqrt{y_0^2 + (v_0/\omega)^2}$ is called amplitude of Motion and angle α or β as phase angle.



Undamped free vibration response.

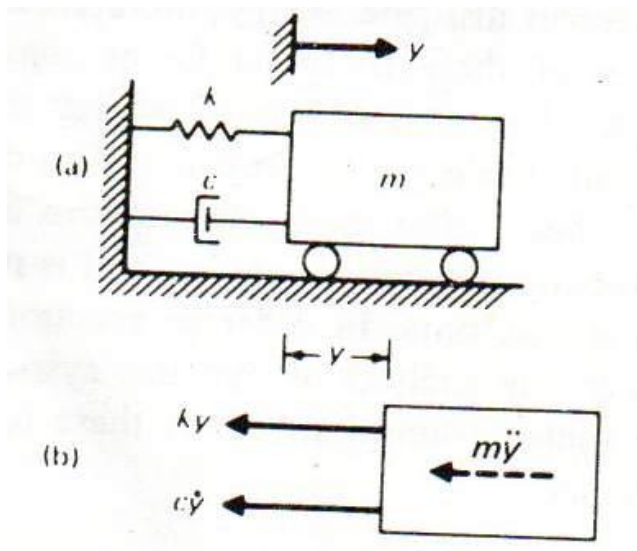
Lecture 2nd

2.1 Viscous Damping:-

In considering damping forces in the dynamic analysis of structures it's usually assumed that these forces are proportional to the Magnitude of the velocity and opposite to the direction of Motion. This type of damping is known as viscous damping. It's the type of damping force that could be developed in a body restricted in its Motion by surrounding fluid.

<u>Equation of Motion of a body with dampers.</u>

Let's assume a modelled structural system as a simple oscillator with viscous damping.



Viscous Damped Oscillator

Where m and k are mass and spring constants of the oscillator. C is the Viscous damping coefficient.

Draw F B D (as show above) and apply Newton's Law to obtain differential equation of Motion.

$\sum Fy = m\ddot{y} + C\dot{y} + Ky = 0$ --------- (1)

In this case $y = A \sin \omega t$ and $y = B \cos \omega t$ will not satisfy (as in case of undamped). However functions $y = Ce^{pt}$ will satisfy subt y in eq --- (1)

$m C p^2 e^{pt} + C Cpe^{pt} + K C e^{pt} = 0$

=> $m p^2 + C P + K = 0$ ------ (2)

p1 and p2 are roots

$= - C/2m \pm \sqrt{(C/2m)^2 - k/m}$ ------ (3)

Thus the general solution of eq (1) is given by super position of the two possible solutions.

$y (t) = C_1 e^{p1t} + C_2 e^{p2t}$ ------- (4)

Where C_1 and C_2 are constants of interrelation and elastic for initial condition.

The final form of eq (4) depends on the sign of expression under radical ----- (3).

Now three cases arise.

$\sqrt{(C/2m)^2 - k/m}$ can be Zero

can be Positive

can be Negative

If $\sqrt{(C/2m)^2 - k/m}$ = Zero, Damping in this case is called critical Damping.

2.2 Critical Damping System

As explained above

$\sqrt{(C/2m)^2 - k/m} = 0$

$(Ccr/2m)^2 - k/m = 0$

$Ccr = 2\sqrt{km}$ ------ (5)

Where Ccr designates critical damping value.

Since Natural frequency of undamped system is given by $\omega = \sqrt{k/m}$ subt in eq ---- (5)

$Ccr = 2m\omega = 2k/\omega$

In a critically damped system the roots of the characteristics eq are equal and eq ---(3)

$p_1 = p_2 = -Ccr/2m$

Since the two roots are equal, the general solution given by eq (4) would provide only one independent constant of Integration hence one independent solution.

$y_1(t) = C_1 e^{-(Ccr/2m)t}$ ------ (6)

Another independent solution may be found by using the subt.

$y_2(t) = C_2 e^{-(Ccr/2m)t}$ ------ (7)

Thus general solution of critically damped system is the given by super position of (6) and (7)

$y(t) = (C_1 + C_2 t) e^{-(Ccr/2m)t}$ ------ (8)

2.3 Over Damped System

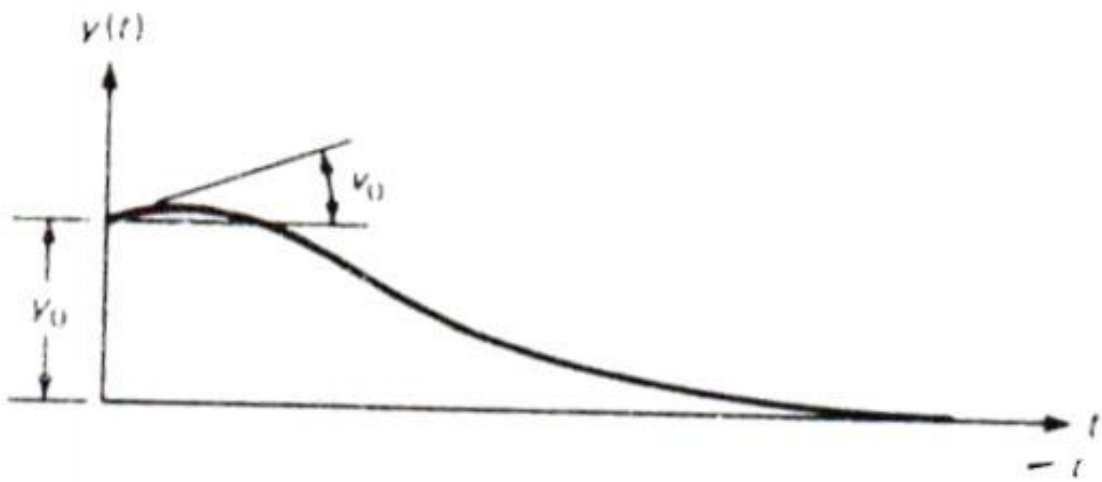
In an over damped system, the damping coefficient is greater than the value for critical damping.

$C > Ccr$

$\sqrt{(C/2m)^2 - k/m}$ is positive and roots are roll and distinct given by

$y(t) = C_1 e^{p1t} + C_2 e^{p2t}$

In case of critical damping and over damping the resulting Motion is not oscillating, the Mag of the oscillation decays exponentially with time to zero.



2.4 Under Damped System

Where the value of damping coefficient is less than critical value ($C < Ccr$)

ie $\sqrt{(C/2m)^2 - k/m}$ is negative < 0

The roots of are complex conjugate

p_1
p_2 $= - C/2m \pm i \sqrt{k/m - (C/2m)^2}$ ------ (9)

Where $i = \sqrt{-1}$ imaginary unit.

For this case make use of Euler's equation.

$e^{ix} = \cos x + i \sin x$

$e^{-ix} = \cos x + i \sin x$

subt eq (9) is roots into

$y(t) = C_1 e^{p1t} + C_2 e^{p2t}$

and use e^{ix} and e^{-ix}

=> $y(t) = e^{-(Ccr/2m)t} (A \cos \omega_D t + B \sin \omega_D t)$ ------ (10)

Where A and B are constants of Integration and ω_D, the damped frequency of system given by

=> $\omega_D = \sqrt{k/m - (C/2m)^2}$

$\omega_D = \omega \sqrt{1-\xi^2}$

We know $\omega^2 = k/m$
or $\omega = \sqrt{k/m}$ } Undamped Natural Frequency

$\xi = C/Ccr$ is damping ratio

Where initial conditions of displacement and velocity are used is $t = 0$ $y = y_0$ and $\acute{y} = v_o$.

A and B can be evaluated eq (10)

$y(t) = e^{-\xi\omega t} [y_0 \cos \omega_D t + (v_o + y_0 \omega/\omega_D) \sin \omega_D t]$ ------ (11)

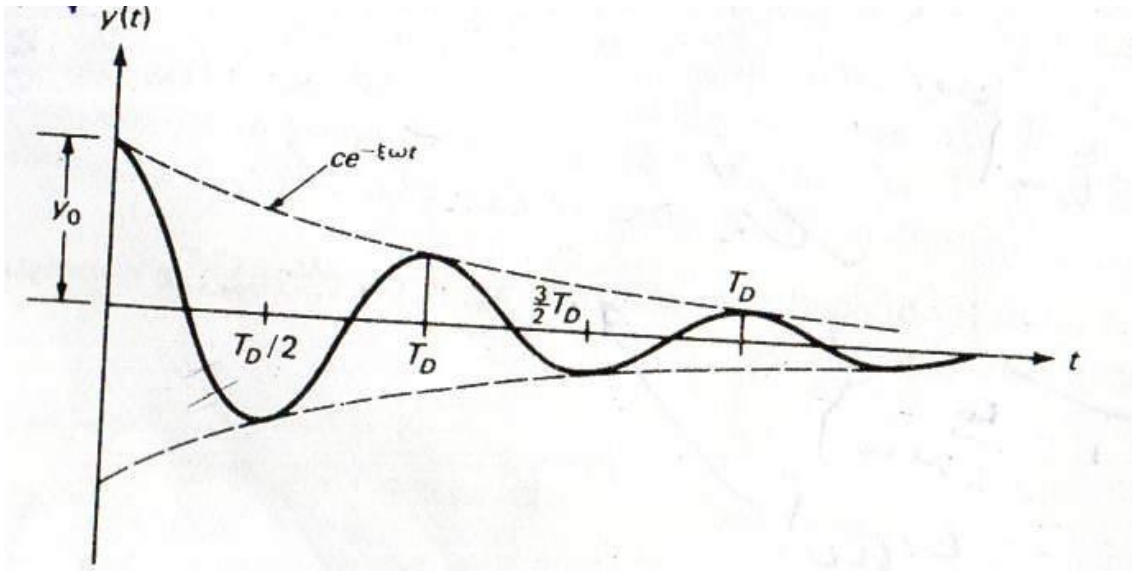
Alternatively

$y(t) = Ce^{-\xi\omega t} (\cos \omega_D t - \alpha)$

$C = \sqrt{y_0^2 + (v_o + y_0 \xi \omega)^2/\omega_D^2}$

$\tan \alpha = (v_o + y_0 \xi \omega)^2/\omega_D y_0$

Graphically under damped system's vibration response is represented by

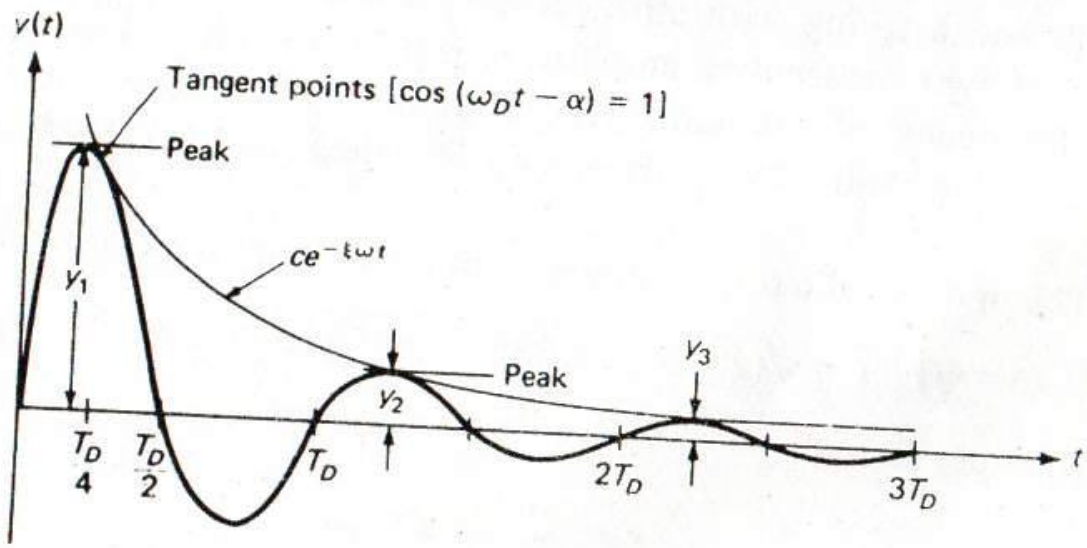


$T_D = 2\pi/\omega_D = 2\pi/\omega\sqrt{1-\xi^2}$

It may be seen Motion is oscillatory but not periodic.

2.5 Logarithmic decrement

A practical method for deterring experimentally, the damping coefficient of a system is to initiate free vibration obtain a record of oscillatory Motion and measure rate of decay of amplitude of Motion.



The decay may be conveniently expressed by log decrement δ ie $\delta = \text{Ln } y_1/y_2$

We know

$y(t) = Ce^{-\xi\omega t} \cos(\omega_D t - \alpha)$

When $\cos(\omega_D t - \alpha) = 1$ ie Tangent pt

$y(t) = Ce^{-\xi\omega t}$

Now

$y_1(t) = Ce^{-\xi\omega t1}$

$y_2(t) = Ce^{-\xi\omega t1}[t_1 + T_D]$

$\delta = \text{Ln } y_1/y_n = \xi\omega T_D$

$T_D = 2\pi/\omega_D$

$\delta = 2\pi\, {}^{\xi}/\sqrt{1-\xi^2}$

Summary

Till now we have learnt about

1. F B D
2. D' Alembert's principle
3. Different eventual equation of undamped simple oscillator ie [SDOF] in free Motion is
   $m\ddot{y} + k\acute{y} = 0$

   $y = A \cos \omega t + B \sin \omega t$
4. Equation of Motion can be written in All forms.

   $y = C \sin [\omega t + \alpha]$

   $y = C \sin [\omega t - \alpha]$

   Where $C \sqrt{y_0^2 + (v_o/\omega)^2}$
5. Differential eq of Motion for damped single degree of freedom system.

   $m\ddot{y} + C\acute{y} + ky = 0$
6. Damping ratio Ccr
   a $\xi = 1$
   b $\xi < 1$
   c $\xi > 1$

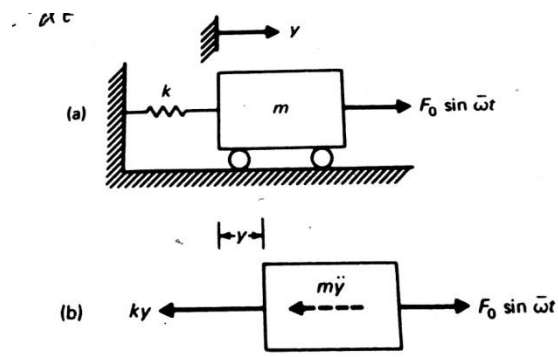
   $y(t) = Ce^{-\xi\omega t} [y_0 \cos \omega_D t + v_o + y_0\xi\, \omega/\omega_D \sin \omega_D t]$
7. Logarithmic decrement.

Lecture 3rd

3.1 Undamped harmonic Excitations

The impressed force F(t) acting on simple oscillator show in fig is assumed to be harmonic and equal to $F_0 \sin \tilde{\omega} t$

Or $F(t) = F_0 \sin \tilde{\omega} t$



Where F_0 is the peak amplitude

$\tilde{\omega}$ = Frequency of force in each/sec

From FBD (fig b)

$m\ddot{y} + Ky = F_0 \sin \tilde{\omega} t$ ------- (1)

$y(t) = y_c(t) + y_p(t)$ ------ (2)

Where $y_c(t)$ is the complementary solution satisfying homogeneous eq. (1) with LHS equal to zero

and yp (t) is particular solution based on solution satisfying the non homogeneous differential eq. (1)

$\therefore\ y_c(t) = A \cos \omega t + B \sin \omega t$ -----(3) $\omega = \sqrt{k/m}$

The nature of forcing function in eq. (1) suggests that particular solution can be taken as

$y_p(t) = Y \sin \omega t$ ----- (4)

Where Y is peak value of particular solution.

Subt eq. (4) in eq (1)

$- m \tilde{\omega}^2 y + ky = F_0$

$y = F_0/k-m \tilde{\omega}^2 = F_0/k/1-r^2$ ------ (5)

$r = \overline{\omega}/\omega$ ------ (6)

frequency ratio r is the ratio of applied forced frequency to the natural frequency of vibration of system

Adding eq (3) and (5)

$y(t) = (A \cos \omega t + B \sin \omega t) + \{(F_0/k)/(1-r^2)\}\sin \tilde{\omega} t$ ------ (6)

When $t = 0$ $y_0 = 0$; $\upsilon_0 = 0$

$A = 0$; $B = rF_0/k/1-r^2$

Subt in eq (6)

$y(t) = F_0/k/1-r^2 (\sin \tilde{\omega} t - r \sin \omega t)$ ----- (7)

From eq (8) we can see the response is given by the superposition of two harmonic terms of different frequencies. The resulting Motion is not harmonic, however in practical case damping forces will always be present in system and will cause free frequency ie 'r sin ωt' to vanish.

Therefore 'r sin ωt' is called transient response.

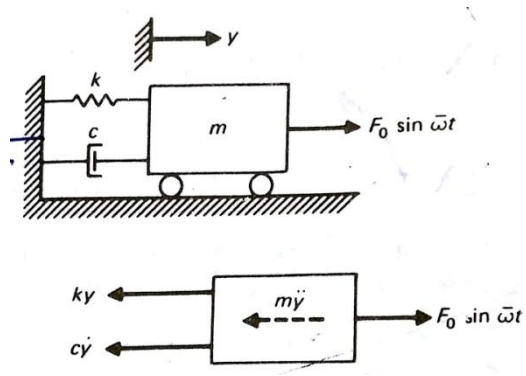
While forcing frequency; $\dot{y}(t) =\{ (F_0/k)/(1-r^2)\}\sin \tilde{\omega} t$ ----- (8) is called steady response.

→If there is no damping transient will not vanish.

→When forcing frequency is equal to Natural frequency (r = 1) the amplitude of Motion becomes large. A system actual upon by an external excitation of frequency coinciding with natural frequency is said to be at resonance.

3.2 Damped harmonic Excitation

Considers the case of one degree of freedom system shown in fig. Vibrating under the influence of viscous damping.



The differential eq of motion is given by

$m\ddot{y} + C\dot{y} + Ky = F_0 \sin \tilde{\omega}\, t$ ------- (9)

The complete solution of this eq is given by

$y(t) = y_c(t) + y_p(t)$

ie complementary solution + particular solution

Complementary solution for under damped case ($C < C_{cr}$) is given as

$y_c(t) = e^{-\xi\omega t}(A \cos \omega_D t + B \sin \omega_D t)$ ------- (10)

The particular solution may be found by substituting y_p in this case assumed to be of the form

$y_p(t) = C_1 \sin\tilde{\omega}t + C_2 \cos\tilde{\omega}t$------- (10)

subst into eq (9) and Equating coefficients of sine and cosine functions.

From Euler's relation

$e^{i\omega t} = \cos\tilde{\omega}t + i\sin\tilde{\omega}t$

We can write eq 9 as

$m\ddot{y} + C\dot{y} + Ky = F_0\, e^{i\omega t}$ ------ (12)

ie $m\ddot{y} + C\dot{y} + Ky = F_0 [\cos \tilde{\omega}t + i \sin \tilde{\omega}t]$

only imaginary component $F_0e^{i\omega t}$ ie force component of $F_0\sin\tilde{\omega}t$ is acting and consequently, the response will them consist only of imaginary part of total solution of eq (12)

In other words total solution has real and imaginary component and disregard real component then particular solution of eq (12) will be of form

$y_p = C^{ie\tilde{\omega}t}$ ------ (13)

subt eq (13) in eq (12)

$-m\,\tilde{\omega}^2 C + iC\tilde{\omega}\,C + kC = F_0$

$C = F_0/k\text{-}m\,\tilde{\omega}^2 + iC\tilde{\omega}$

$y_p = F_0\, e^{i\tilde{\omega}t}/\, k\text{-}m\,\tilde{\omega}^2 + iC\tilde{\omega}$ ------ (14)

By using polar coordinate form of complete denominate in eq (14)

$y_p = F_0\, e^{i\tilde{\omega}t} /\sqrt{(k\text{-}m\,\tilde{\omega}^2)^2 + (C\tilde{\omega})^2}\, e^{i\theta}$

Or $y_p = F_0\, e^{i(\tilde{\omega}t-\theta)}/\sqrt{(k\text{-}m\,\tilde{\omega}^2)^2 + (C\tilde{\omega})^2}$ ------ (15)

$\tan\theta = C\tilde{\omega}/k\text{-}m\tilde{\omega}^2$ ------ (16)

The response to the force $F_0\sin\tilde{\omega}t$ ie imaginary component of $F_0e^{i\tilde{\omega}t}$ imaginary component of eq (15)

$y_p = F_0 \sin(\tilde{\omega}t-\theta)/\sqrt{(k\text{-}m\,\tilde{\omega}^2)^2 + (C\tilde{\omega})^2}$ ------ (17)

$y_p = y \sin(\tilde{\omega}t - \theta)$ ------ (18)

$y = F_0/\sqrt{(k\text{-}m\,\tilde{\omega}^2)^2 + (C\tilde{\omega})^2}$ is the amplitude of steady state Motion

Eq (17) and (18) may be written as

$y_p = y_{st} \sin(\omega t - \theta)\sqrt{(1-r^2)^2 + (2\xi r)^2}$ ------ (19)

$\tan\theta = 2\xi r/1\text{-}r^2$ ------ (20)

Where $y_{st} = F_0/k$, static deflection of spring acted upon by force F_0

$\xi = C/C_{cr}$, damping ratio

$r = \tilde{\omega}/\omega$, frequency ratio

Total response of Damped Harmonic Excitation is given by

$y(t) = e^{-\xi\omega t} (A \cos \omega_{Dt} + B \sin \omega_{Dt}) + y_{st} \sin (\tilde{\omega}t - \theta)/\sqrt{(1-r^2)^2 + (2\xi r)^2}$ ------ (21)

Dynamic Magnification factor

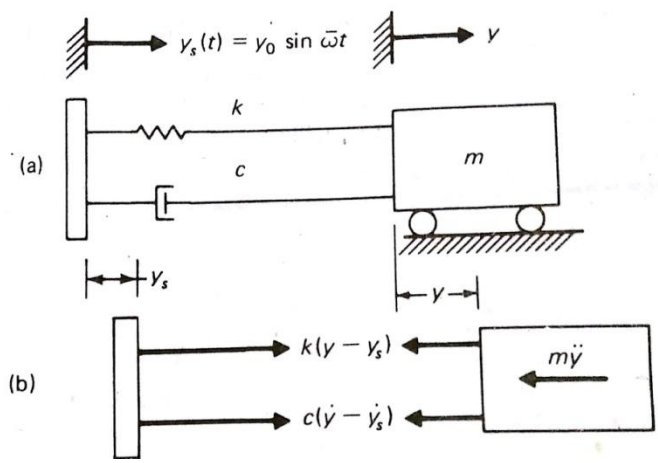
$D = y/y_{st} = 1/\sqrt{(1-r^2)^2 + (2\xi r)^2}$ ------ (22)

D varies with frequency ratio r and damping ratio ξ.

3.3.1Response to support Motion

There are many actual cases where the foundation or support of a structure is subjected to time varying motion structures subjected to ground motion by earthquakes or other excitations such as explosion or dynamic action of machinery are examples in which support motions may have to be considered in the analysis of dynamic response.

Let's consider a simple oscillator as shown in fig.



$y_s(t) = y_0 \sin\tilde{\omega}t$ ------ (23)

Where y_0 is maximum amplitude

$\tilde{\omega}$ frequency of the support

From F. B.D (fig. b)

Differential eq of Motion in horizontal direction

$m\ddot{y} + C(\dot{y}-\dot{y}_s) + K(y-y_s)= 0$ ------ (24)

subt eq (23) into eq (24)

$m\ddot{y} + C\dot{y} + Ky + Ky_0 \sin \tilde{\omega}t + C\,\tilde{\omega}\, y_0 \cos \tilde{\omega}t$ ------- (25)

$m\ddot{y} + C\dot{y} + Ky = F_0 \sin (\tilde{\omega}t + \beta)$ ------ (26)

Where $F_0 = y_0 \sqrt{k + (C\tilde{\omega})^2}$

$F_0 = y_0 k\sqrt{1 + (2r\xi)^2}$ ------ (27)

$\tan \beta = C\tilde{\omega}/k = 2r\xi$ ------ (28)

Eq (26) is the differential equation for oscillator excitation by harmonic force $F_0 \sin(\tilde{\omega}t + \beta)$ and is of same force as eq of damped harmonic Excitation.

$y (t) = F_0/k \sin (\tilde{\omega}t + \beta - \theta)/\sqrt{(1-r^2)^2 + (2r\xi)^2}$ ------ (29)

subt F_0 from eq (27)

$y (t)/y_0 = \{\sqrt{1 + (2r\xi)^2}/\sqrt{(1-r^2)^2 + (2r\xi)^2}\}\sin (\tilde{\omega}t + \beta - \theta)$ ------ (30)

Eq (30) is the expression for the relative transmission of the support motion to the oscillator. This is an important problem in vibration isolation in which equipment must be protected from harmful vibration of supporting structure. The degree of relative isolation is known as transmissibility.

3.3.2 Transmissibility is defined as ratio of Motion Y of oscillator to amplitude y_0 the Motion of support

From eq 30, transmissibility $Tr = y/y_0 = \sqrt{1 + (2r\xi)^2}/\sqrt{(1-r^2)^2 + (2r\xi)^2}$------- (31)

Damping tends to reduce the effectiveness of vibration isolation for frequencies ie frequency ratio greater than $\sqrt{2}$ ie $r > \sqrt{2}$

3.3.3 Force Transmitted to the Foundation

Consider a damped oscillator with a harmonic Force $F(t) = F_0 \sin\tilde{\omega}t$ acting on its Mass as shown in last fig.

The differential eq of Motion is

$m\ddot{y} + C\dot{y} + Ky = F_0 \sin \tilde{\omega}t$

with steady state solution

$y = Y \sin (\omega t-\theta)$

Where $Y = F_0/k/\sqrt{(1-r^2)^2 + (2r\xi)^2}$

$\dot{y} = Y \tilde{\omega} \cos (\tilde{\omega}t-\theta)$

and $\tan \theta = 2r\xi/1-r^2$

The force transmitted to the support through spring ky and through damping element is $C\dot{y}$. The total force transmitted F_T

$F_T = ky + C\dot{y}$

$F_T = Y [k \sin (\tilde{\omega}t-\theta) + C\tilde{\omega} \cos(\tilde{\omega}t-\theta)]$

$F_T = Y \sqrt{k^2+C^2 \tilde{\omega}^2} \sin (\tilde{\omega}t-\theta+\beta)$

$F_T = Y \sqrt{k^2+C^2 \tilde{\omega}^2} \sin (\tilde{\omega}t+\phi)$ ---- Total phase angle

in which $\tan \beta = C\tilde{\omega}/k = 2\xi^r$ and $\phi = \theta-\beta$

From value of Y and F_T, the force A_T transmitted to the foundation is

$A_T = F_0 \sqrt{1 + (2r\xi)^2/(1-r^2)^2 + (2r\xi)^2}$

Transmissibility; $T_r = A_T/F_0 = \sqrt{1 + (2r\xi)^2/(1-r^2)^2 + (2r\xi)^2}$

Transmissibility of Motion from foundation to the structure and transmissibility of force from structure to foundation are given by exactly same function.

We know $\phi = (\theta-\beta)$

$\tan \theta = \tan \theta -\tan \beta/1 + \tan \theta \tan \beta$

$\tan \theta = 2\xi r/1-r^2$; $\tan \beta = 2\xi r\tan \phi = 2\xi r/1-r^2 + 4\xi^2r^2$

Note:- Module to be continued

For Queries

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